

Energy and momentum deposited into a QCD medium by a jet shower

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(Dated: September 8, 2009)

Hard partons moving through a dense QCD medium lose energy by radiative emissions and elastic scatterings. Deposition of the radiative contribution into the medium requires rescattering of the radiated gluons. We compute the total energy loss and its deposition into the medium self-consistently within the same formalism, assuming perturbative interaction between probe and medium. The same transport coefficients that control energy loss of the hard parton determine how the energy is deposited into the medium; this allows a parameter free calculation of the latter once the former have been computed or extracted from experimental energy loss data. We compute them for a perturbative medium in hard thermal loop (HTL) approximation. Assuming that the deposited energy-momentum is equilibrated after a short relaxation time, we compute the medium's hydrodynamical response and obtain a conical pattern that is strongly enhanced by showering.

PACS numbers: 12.38.Mh, 11.10.Wx, 25.75.Dw

Jet quenching (the modification of hard jets in dense media) is one of the most studied discoveries at the Relativistic Heavy-Ion Collider (RHIC) [1]. It is expected to play a key role in the study of the quark-gluon plasma (QGP) produced in heavy-ion collisions at the Large Hadron Collider (LHC). Numerous experiments [2] have established the suppression of hadrons with high transverse momenta; others indicate that the lost energy manifests itself as conical flow in the soft sector [3].

Calculations of jet modification tend to focus on one of two separate questions: the modification of the final hadron distribution from the hard parton due to its *energy loss*, or the response of the medium to the *energy deposited*. Numerous studies of the former, based on perturbative QCD (pQCD), have yielded near-rigorous measures of the two non-perturbative transport coefficients $\hat{q} = \frac{dp^2}{dL}$ and $\hat{e} = \frac{dE}{dL}$ which codify the transverse (to the jet axis) momentum diffusion and longitudinal drag experienced by a fast parton [4]. Computations of the medium response consist of two parts: an *ansatz* for the space-time profile of the energy-momentum deposition, and a calculation of the dynamical response to this “source” of excess energy and momentum. Based on its success at RHIC, ideal fluid dynamics has been used to compute this medium response [5], assuming that the energy lost by the jet is entirely deposited into the medium at a constant rate and thermalizes instantaneously.

So far there exists no first principles calculation of the magnitude and space-time profile of the energy-momentum *deposited* in a medium by a hard parton that can be considered on par with the pQCD *energy loss* calculations [6]. A noteworthy attempt to calculate the deposition profile in pQCD is the semi-phenomenological approach of Neufeld and Müller [7] who use the differential single gluon emission spectrum of Ref. [8] and interpret this as the rate of gluon emission in the medium. A non-diffusive Fokker-Planck equation is then motivated to compute how this distribution changes due to elastic

energy loss of the emitted gluons. As anticipated in [9], they find that not all of the energy lost to gluon radiation is deposited in the medium. However, since the underlying formalism [8] lacks information about virtuality evolution, this calculation does not include gluon multiplication by showering, *i.e.* the splitting of a radiated gluon into two lower virtuality gluons. The transverse momentum deposition thus cannot be computed and, due to the strict eikonal limit used in [8], the parent parton does not lose energy after radiation. We present a new formalism in which the radiative and elastic energy loss of the fast parton, its virtuality evolution by radiation, the showering and multiplication of the radiated gluons, and the energy deposited by them in the medium are all calculated consistently in the same approach.

Hard jets in vacuum or in heavy ion collisions are produced with considerable virtuality. As the jets proceed through vacuum or medium, this virtuality is lost by sequential radiative emissions. The effect of this perturbative shower on the non-perturbative hadronization process is computed using DGLAP evolution equations [10] for the fragmentation function. These equations express the radiation of multiple partons, which hadronize independently, via an evolution in virtuality of the parent parton. In a medium, one can derive analogous equations where the gluon radiation probability is modified by the scattering of hard parton and emitted gluons off medium constituents. These are referred to as “medium modified evolution equations”. In addition to stimulating gluon emission, the scattering of the hard parton causes it to lose forward light-cone momentum by elastic exchanges with the medium [11, 12]. At the same time the parton gains transverse momentum from the medium [13] and imparts to it an equal amount in return. In an arbitrary medium these effects are encoded in two non-perturbative transport coefficients, \hat{q} and \hat{e} , defined in terms of in-medium gluon field correlation functions [12, 13]. The medium modification of the standard vac-

uum evolution depends on these transport coefficients.

In-medium evolution equations where the medium modified fragmentation function (MMFF) is affected only by \hat{q} were derived in [14]. We point out that the same processes can be used to compute the amplification of the energy deposited through multiple radiations stimulated by transverse broadening. Formally, this can be computed by replacing the operator expression for the fragmentation function with that for the energy deposited; this is identical to \hat{e} . Using this calculation of the modified \hat{e} as the energy deposited through all elastic scatterings of the shower places it on the same footing as energy loss. The diagrams involved and the resulting expressions for the in-medium splitting functions (IMSF) are identical. The solution of the in-medium evolution equation for \hat{e} no longer represents the elastic energy loss by one parton, but rather the energy deposited by the jet shower.

Imagine a hard quark or gluon with large light cone momentum q^- (and thus energy $E = q^-/\sqrt{2}$) and virtuality $\leq \mu$ entering a medium of fixed length L held at a constant temperature T . Let us assume that the rate of energy deposition by this jet in the medium as a function of length ζ , denoted as $\frac{d\Delta E}{d\zeta}(E, \zeta, \mu^2)$, is known (i.e., can be calculated or measured). Note that both the deposited energy ΔE and ζ are actually the light-cone quantities Δq^- and ζ^- . For brevity we refer to these simply as deposited energy and distance travelled. Given the above function, the total energy deposited by a jet originating at location ζ_i and propagating to ζ_f is given as

$$\Delta E(E, \mu^2)_{\zeta_i}^{\zeta_f} = \int_{\zeta_i}^{\zeta_f} d\zeta \frac{d\Delta E}{d\zeta}(E, \zeta, \mu^2) \stackrel{1 \text{ parton}}{\simeq} (\zeta_f - \zeta_i) \hat{e}, \quad (1)$$

where the last approximate equality is solely for the case of a single parton propagating without radiation.

If the scale μ is much larger than Λ_{QCD} , the change with virtuality in the partonic shower pattern may be calculated perturbatively: a leading quark at the higher virtuality may split into a quark and a gluon with lower virtuality, and similarly for a gluon. As a result, there is change in the energy deposited in the medium due to the increase of the number of partons depositing energy. Using the IMSF from [14], the change in the energy deposition by a quark with energy E from ζ_i to ζ_f due to the increase in virtuality μ can be expressed as [15]

$$\begin{aligned} \frac{d\Delta E_q(E, \mu^2)_{\zeta_i}^{\zeta_f}}{d\ln(\mu^2)} &= \frac{\alpha_s(\mu^2)}{2\pi} \int_0^1 dy \int_{\zeta_i}^{\zeta_f} d\zeta P_{q \rightarrow qg}(y, \zeta, \mu^2, E) \\ &\times \left[\Delta E_q(E, \mu^2)_{\zeta_i}^{\zeta} + \Delta E_q(yE, \mu^2)_{\zeta}^{\zeta_f} + \Delta E_g((1-y)E, \mu^2)_{\zeta}^{\zeta_f} \right]. \end{aligned} \quad (2)$$

Here the first term in square brackets represents the energy deposited by a quark with energy E and virtuality μ^2 , from the initial location ζ_i to the intermediate location ζ ; the second and third terms represent the energy deposited by the quark and the emitted gluon with reduced energies yE and $(1-y)E$, respectively, from the

intermediate location ζ to the final location ζ_f . In Eq. (2) the quark IMSF $P_{q \rightarrow qg}$ is given as [14, 16]

$$P_{q \rightarrow qg} = \frac{\hat{q} C_F}{2\pi\mu^2} \frac{1+y^2}{1-y} \left[2 - 2 \cos \left(\frac{\mu^2 \zeta}{2Ey(1-y)} \right) \right]. \quad (3)$$

The increase in the energy deposited due to the splitting of the parton is reduced by the virtual correction which restores unitarity to the evolution equations. The effect of such corrections on Eq. (2) is incorporated by subtracting from it the virtual term

$$V = \frac{\alpha_s(\mu^2)}{2\pi} \Delta E_q(E, \mu^2)_{\zeta_i}^{\zeta_f} \int_0^1 dy \int_{\zeta_i}^{\zeta_f} d\zeta P_{q \rightarrow qg}(y, \zeta, \mu^2, E). \quad (4)$$

Along with the energy deposition from a quark jet one has to evolve the one from a gluon jet of virtuality $\leq \mu$, using a similar evolution equation that includes the splitting of a gluon into two gluons or a $q\bar{q}$ pair. Similar to the MMFFs, one solves a coupled set of evolution equations for $\Delta E_q(E, \mu^2)_{\zeta_i}^{\zeta_f}$ and $\Delta E_g(E, \mu^2)_{\zeta_i}^{\zeta_f}$ both of which are functions of three variables E, ζ_i, ζ_f at the scale μ^2 .

The evolution equations (2,3,4) for a quark jet and the coupled equations for gluon jets are motivated by existing rigorous derivations of the medium modification of the fragmentation functions due to gluon emission [16], the accumulation of transverse momentum [13] and longitudinal drag [12] by propagating hard partons in a QCD medium, and the effect of such accumulated momentum on radiative processes [17]. The IMSF (3) accounts for interference between diagrams where the gluon is emitted at the origin or at the location ζ . In propagating up to ζ the quark loses a fraction of its energy; while this is included in the total energy deposited, its effect on the interference pattern in Eq. (3) is ignored; this is justified in the eikonal limit for the propagating parton. Yet another approximation is the neglect of the energy lost by the radiated (reabsorbed) gluon in the virtual correction. Since the radiated gluon in the virtual correction exists in only one amplitude, with a single parton in the complex conjugate, its energy loss is balanced by the quark propagating in the loop.

In the eikonal approximation, the hard jet loses light cone momentum and remains close to on-shell, thus the z -component of the deposited light-cone momentum is approximately equal to the energy deposited ($\Delta p_z \simeq \Delta E$). Note that the negative light cone momentum (Δq^-) is not conjugate to ζ^- and thus it is not inconsistent to compute the ζ^- dependence of the Δq^- deposited. The remaining two components that may be computed are the transverse momentum deposited by the jet as a function of ζ^- . This can again be directly estimated from a pQCD calculation: A parton traversing a medium gains transverse momentum squared with length as

$$\langle p_\perp^2 \rangle(E, \mu^2)_{\zeta_i}^{\zeta_f} = \int_{\zeta_i}^{\zeta_f} d\zeta \frac{d\langle p_\perp^2 \rangle(E, \mu^2)}{d\zeta} \stackrel{1 \text{ parton}}{\simeq} (\zeta_f - \zeta_i) \hat{q}. \quad (5)$$

By momentum conservation this equals the p_\perp^2 deposited in the medium by the same parton.

For a hard virtual quark the total transverse momentum deposited increases due to parton splitting. This can be calculated using an equation similar to that for light-cone momentum deposition. For a quark with energy E and virtuality μ^2 , traversing a medium from ζ_i to ζ_f , the change of the transverse momentum deposited with virtuality is obtained as

$$\frac{d\langle p_\perp^2 \rangle_q(E, \mu^2)_{\zeta_i}^{\zeta_f}}{d\ln(\mu^2)} = \frac{\alpha_s(\mu^2)}{2\pi} \int_0^1 dy \int_{\zeta_i}^{\zeta_f} d\zeta P_{q \rightarrow qg}(y, \zeta, \mu^2, E) \quad (6)$$

$$\times \left[\langle p_\perp^2 \rangle_q(E, \mu^2)_{\zeta_i}^{\zeta_f} + \langle p_\perp^2 \rangle_q(yE, \mu^2)_{\zeta}^{\zeta_f} + \langle p_\perp^2 \rangle_g((1-y)E, \mu^2)_{\zeta}^{\zeta_f} \right].$$

The splitting function here is identical to that in Eq. (3), and the meaning of the three terms in the bracket is analogous to Eq. (2). Further, one must include a virtual correction and couple Eq. (6) to a similar equation for the p_\perp^2 deposited by a virtual gluon.

Using Eqs. (2,6) (along with the coupled ones for gluon jets), we can compute the 3-momentum Δq^- , \vec{p}_T deposited by a hard virtual parton, disintegrating into a shower of partons, in a dense medium as a function of the length ζ^- traversed. Similar to the case of in-medium evolution equations for the MMFF [14], these equations require an initial condition. For the case of the MMFF, the only possible choice was to insist that the part of the jet with virtuality below a minimum μ_0^2 exited the medium and use the known vacuum FF at that scale as an input. For the deposited part of the energy-momentum we here assume that the medium is weakly coupled, thus when the virtuality of the parton is $\mu_0 \simeq 4T$ the deposited energy and p_\perp^2 can be obtained from the expressions for \hat{e} and \hat{q} in an HTL plasma [11]:

$$d\Delta E(\mu_0, E)/d\zeta = C_R \alpha_s(\mu_0^2) m_D^2 \log \left[(4ET/m_D^2)^{1/4} \right],$$

$$d\langle p_\perp^2 \rangle(\mu_0, E)/d\zeta = C_R \alpha_s(\mu_0^2) T m_D^2 \log \left[4ET/m_D^2 \right]. \quad (7)$$

Here m_D is the Debye screening length and C_R is the representation specific Casimir. The integrated energy and p_\perp^2 deposited from Eq. (7), as a function of the length traveled, is plotted for gluons (circled) and quarks as solid lines in Figs. 1 and 2. For a consistent description, we impose that partons with an energy $E < 4T$ become part of the thermal medium. This condition is maintained through the evolution equations.

Using Eq. (7) as input, we may calculate the increase in the energy and p_\perp^2 deposition in the medium as a function of ζ for initially highly virtual hard partons that evolve into a radiative shower. Starting from the scale of $\mu_0 = 4T$ (in all calculations we pick $T = 300$ MeV and a partonic plasma with 3 quark flavors) we evolve up to an initial scale $\mu = E/2$. These are plotted as dashed lines in Figs. 1 and 2. One notes immediately that both quantities increase as we evolve up in virtuality. For comparison, we also estimate the total energy lost by the

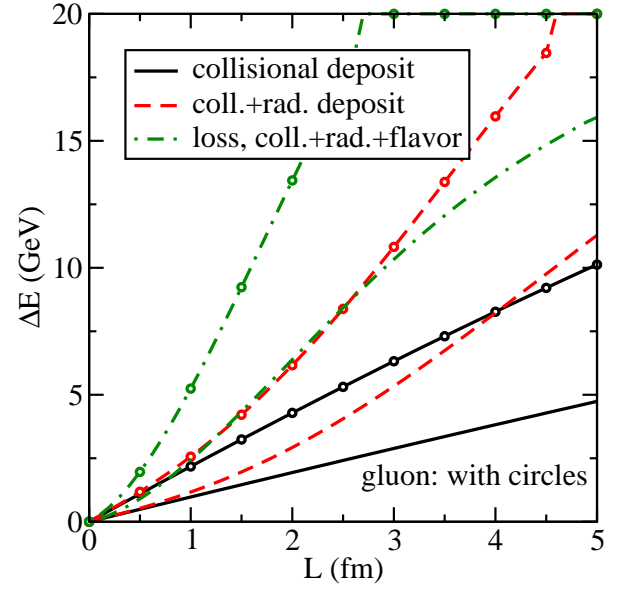


FIG. 1: (Color online) Dash-dotted: Total energy lost by a hard gluon (circled) or quark by radiative, elastic and flavor changing processes. Solid: Energy deposited in the medium by a hard parton which does not radiate. Dashed: The same for a virtual parton devolving into a partonic shower.

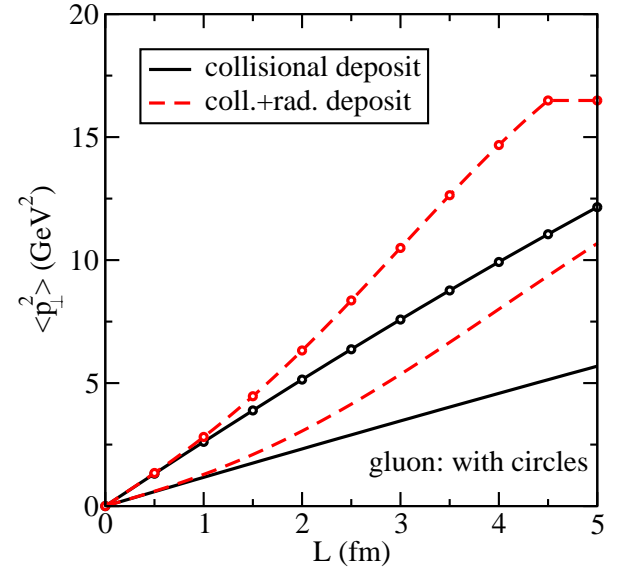


FIG. 2: (Color online) The $\langle p_\perp^2 \rangle$ deposited by a hard gluon (circled) or quark without radiative emission (solid) and with a full radiative shower (dashed).

hard parton due to elastic, radiative inelastic and flavor changing interactions (dash-dotted lines in Fig. 1). The last type of energy loss refers to the case where a quark splits with the gluon carrying a larger fraction of the momentum, or a gluon splits into a quark-antiquark; in this case we assume that the entire energy of that parent parton has been lost. This leads to a somewhat artificial enhancement of the total energy loss.

As an illustration of the effect of this energy-momentum deposition in the medium, we compute its hydrodynamic response to the following source term:

$$J^\mu \equiv \left[\frac{d\Delta E(\mu, E)}{d\zeta}, 0, 0, \frac{dp_z(\mu, E)}{d\zeta} \right] \delta^2(\vec{r}_\perp) \delta(t-z). \quad (8)$$

In this first attempt we ignore the transverse momentum contribution to the source current. Following Refs. [18], we assume that the energy deposited is a small perturbation and solve for the linear response of the medium:

$$T^{\mu\nu} \simeq T_0^{\mu\nu} + \delta T^{\mu\nu}; \quad \partial_\mu T_0^{\mu\nu} = 0, \quad \partial_\mu \delta T^{\mu\nu} = J^\nu. \quad (9)$$

$T_0^{\mu\nu}$ is the unperturbed energy-momentum tensor of a homogeneous and static partonic medium in equilibrium. The small excess $\delta T^{\mu\nu}$ is decomposed as

$$\begin{aligned} \delta T^{00} &\equiv \delta\epsilon, & \delta T^{0i} &\equiv g^i, \\ \delta T^{ij} &= \delta_{ij} c_s^2 \delta\epsilon - \Gamma_s (\partial^i g^j + \partial^j g^i - \frac{2}{3} \delta_{ij} \nabla \cdot \vec{g}). \end{aligned} \quad (10)$$

$\delta\epsilon$ is the excess energy density, \vec{g} is the momentum current density and $\Gamma_s = \frac{\eta}{sT}$ is the sound attenuation length. For the specific shear viscosity we took $\frac{\eta}{s} = \frac{1}{2\pi}$. We delay the response to the source J by a time $\tau_{\text{rel}} = \frac{1}{m_D}$ to account for thermalization of the deposited energy.

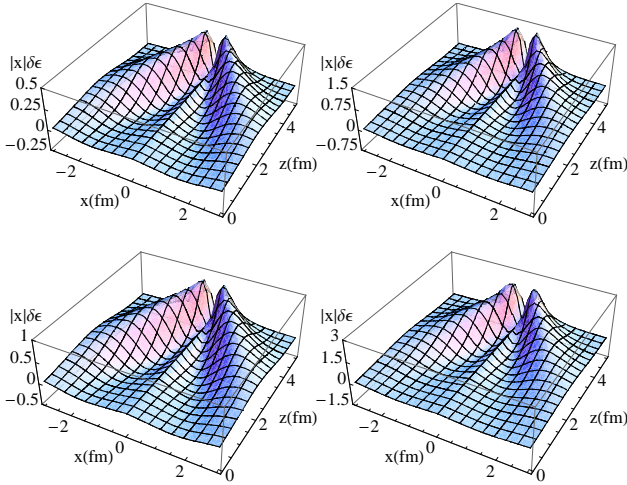


FIG. 3: (Color online) The linear fluid dynamical response to the energy deposited by a single parton (left) or by a parton-initiated shower (right), when the parton is a quark (top) or a gluon (bottom). Note the different vertical scales.

In Fig. 3 we show the azimuthal projection of the energy density $|x|\delta\epsilon$ at $t = 5\text{fm}/c$ after the parton is created, for a single non-radiating parton (left) and a parton-initiated jet shower (right). A gluon (bottom row) deposits more energy than a quark (top row), due to its larger color factor that enters both in the elastic energy loss and shower production rate. One immediately notes that, while the basic Mach cone structure is not changed, showering leads to an enhancement by a factor of 3 in the

overall magnitude of the response. For quark jets our results are qualitatively similar to Ref. [7].

In this Letter, we have presented a consistent pQCD based calculation of the light-cone and transverse momentum ($\Delta q^-, p_T^2$) deposited by a jet in a medium, as a function of distance traversed. Assuming a short thermalization time for the deposited energy we also computed the hydrodynamic response. The pQCD shower has the effect of a large part of the energy being deposited later in the history of the jet [7] which tends to enhance the Mach cone like structure formed.

We thank B. Müller and R. B. Neufeld for helpful discussions. This work was supported by the U.S. Department of Energy under grant DE-FG02-01ER41190.

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